The contribution M_3 can be obtained in a similar manner, using the values of $P_2(S_\lambda)$, $P_1(S_\lambda)$, $P_2'(S_\lambda)$, $P_1'(S_\lambda)$, M_2, and M_1:

$$M_3 = \frac{\mu J_3 R_E^3}{6R^6}\Big\{\big[315(r\cdot n)^2 - 45\big](n\times \underline{I}\cdot r + r\times \underline{I}\cdot n) + \big[315(r\cdot n) - 945(r\cdot n)^3\big] r\times \underline{I}\cdot r - 90(r\cdot n) n\times \underline{I}\cdot n\Big\} \quad (41)$$

Conclusions

A recursive vector-dyadic expression for the contribution of a zonal harmonic of degree n to the gravitational moment about the mass center of a small body can be obtained by a procedure that involves differentiating a celestial body's gravitational potential twice with respect to a vector. The recursive property of the result is a consequence of taking advantage of a recursion relation for Legendre polynomials that appear in the gravitational potential. When a celestial body's gravitational potential includes zonal harmonics, the preceding vector-dyadic expression is useful for calculating their contributions to the gravitational moment. The contribution of the zonal harmonic of degree 2 is consistent with the gravitational moment exerted by an oblate spheroid.

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Satellite Relocation by Tether Deployment

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Introduction

ONE of the limitations of the working lifetime of a satellite is the expenditure of fuel required for changing and maintaining the orbit. In geosynchronous orbit, the orbit is subject to perturbations, primarily due to the gravitational effects of the sun and the moon. Fuel is also required for relocating a satellite, for example, if a synchronous satellite positioned over Indonesia is desired to be relocated over South America. We propose that a satellite can be repositioned in orbit without fuel expenditure by extending a mass on the end of a tether. The tether may also serve other purposes, including energy storage for eclipse power.

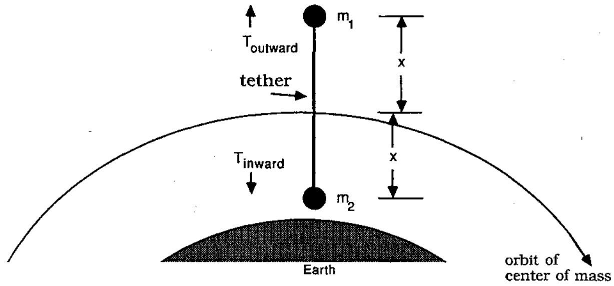

Fig. 1 Tether orbit and definitions.

A tether is a long, flexible cable that connects one part of a satellite with another. Tethers have recently been well covered in the aerospace literature.[1-5] In the equilibrium configuration, as shown in Fig. 1, the tether is oriented radially outward, with a tension on the tether due to the gravitational gradient (or "tidal") force.

Mass of the cable is an important figure. A figure of merit for material strength is the critical length L_c, the length of untapered cable that could be suspended in a 1-g gravitational field. One proposed cable material is Kevlar. For one common type of Kevlar, the critical length is 250 km.[6] The effective acceleration due to the gravity gradient a distance x from the center of mass (cm) is to first order

$$a_{\rm eff} = 3g\, r_e^2 x / r_o^3 \quad (1)$$

where r_o is the orbital radius and r_e is the radius of the Earth. At geosynchronous Earth orbit (GEO), $3\, r_e^2/r_o{}^3 = 1.6\cdot 10^{-6}$ km^{-1}. The minimum mass m_t of untapered cable required to support an end mass m_o is thus to first order (for x in km):

$$m_t\,(\text{GEO}) = 1.6\cdot 10^{-6}\cdot m_o x^2/L_c \quad (2)$$

Thus, in GEO a 1000-km-long tether can easily be made much less massive than the satellite it supports.

Most analyses of tether orbits assume that the center of mass of a tethered satellite system remains in the original orbit,[1,2] i.e., that the angular velocity of the tethered satellite does not change as the tether is extended or retracted. We note that this is true only to the first-order approximation in tether length. Briefly, the mass that extends outward experiences an increase in centrifugal force that increases linearly with distance, but the mass that extends inward experiences an increase in gravity that increases faster than the linear increase. Thus, the center of mass of the orbit is pulled inward, and to conserve angular momentum, the angular velocity of the orbit increases.

Mathematical Analysis

In the following discussion, we shall assume a tether of negligible mass in circular orbit. The extension of the analysis to tethers of nonnegligible mass is straightforward.

Consider a satellite of mass m_i consisting of two pieces of masses m_1 and m_2 connected by a tether. For calculation simplicity, let $m_1 = m_2 = m_i/2$. The initial orbit is assumed to be circular, with an angular velocity ω_o and an initial orbital radius (measured from the Earth's center) r_o. With the tether at initial length zero, the orbit has initial angular momentum

$$L_i = m_i \omega_o r_o^2 \quad (3)$$

Received April 6, 1989; revision received June 15, 1989.

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Force balance (outward centrifugal force equals inward gravitational force) requires $\omega_o^2 r_o^3 = GM$, where GM is the gravitational constant times the mass of the Earth. Now assume that the tether is extended to length x in each direction from the center of mass, as shown in Fig. 1. The total length is $2x$. Note that energy is not conserved, since in deploying a tether, work is done by the gravity gradient force a_{eff}. Angular momentum is still conserved

$$L = m_1 \omega r_1^2 + m_2 \omega r_2^2 \tag{4}$$

where r_{cm} is the orbital radius of the center of mass, and $r_1 = r_{\text{cm}} - x$ and $r_2 = r_{\text{cm}} + x$. The system must also have a force balance, which implies (for a massless tether) that the inward tension on the low end of the tether equals the outward tension on the high end of the tether,

$$\omega^2(r_1 + r_2) = GM \cdot (r_1^{-2} + r_2^{-2}) \tag{5}$$

If we expand this to second order in x, then set Eq. (3) equal to Eq. (4) to solve Eq. (5) for ω and r_{cm} as a function of tether extension x, we find

$$r_{\text{cm}} = r_o - 5x^2/r_o \tag{6}$$

and the orbital period P is

$$P = P_o\,[1 - 9(x/r_o)^2] \tag{7}$$

The result is that the orbital period decreases as the tether length is extended. For example, a GEO satellite consisting of two equal masses on a 1000-km-long tether will have a period faster than that of an untethered satellite by 0.44 deg per day. Inclusion of higher-order terms results in an increase in the effect.

If the two masses are allowed to differ, the orbital period change is proportional to $m_1 m_2/(m_1 + m_2)$, which is maximum at $m_1 = m_2 = m_t/2$.

Satellite Relocation

We propose to use this effect to relocate a satellite without expenditure of fuel. Consider a satellite in orbit that consists of two masses connected by a tether with a motorized spool that allows it to be pulled out or in. Preferably the spool contains a motor/generator, such that the energy used pulling the tether in can be regained (except for friction losses) when the tether is let out. It is also important that the spool have a controller that can be programmed to damp lateral oscillations in the tether during the reeling process.

The spool motor can be powered by the satellite solar array, which does not expend fuel.

Pulling the tether in will result in a longer orbital period for the satellite, and thus drift the satellite backward (west) in longitude; whereas letting the tether out will result in a shorter orbital period and thus drift the satellite forward (east). This is shown schematically in Fig. 2.

Maximum relocation capability is achieved when the masses on both ends of the tether are equal. The countermass could be the spent booster used to boost the satellite up from low Earth orbit or the apogee kick motor. Alternatively, the countermass could be systems that do not require to be located in close proximity to the lower part of the satellite or could be an entirely separate satellite system. Finally, if the tether is sufficiently long, no counterweight would be needed, with the mass of the tether itself providing this function. In this case a single tether could extend either inward or outward.

In the normal operating position, the tether would be extended to a length $1/\sqrt{2}$ of full length (about 0.707). If the tether is 1000 km long, this allows the satellite to be moved forward as fast as 0.22 deg/day by fully extending the tether, or to lag backward up to 0.22 deg/day by fully retracting it. Thus, relocation by a few degrees in longitude could be accomplished in a week or so, and the worst possible case, 180 deg relocation, in 2 years.

The preceding analysis has assumed equilibrium conditions, i.e., that the orbit remains circular during the extension and deployment of the tether. This assumption is true only if the tether is deployed or retracted over a time greater than an orbital period. Faster deployment results in changes to the orbital

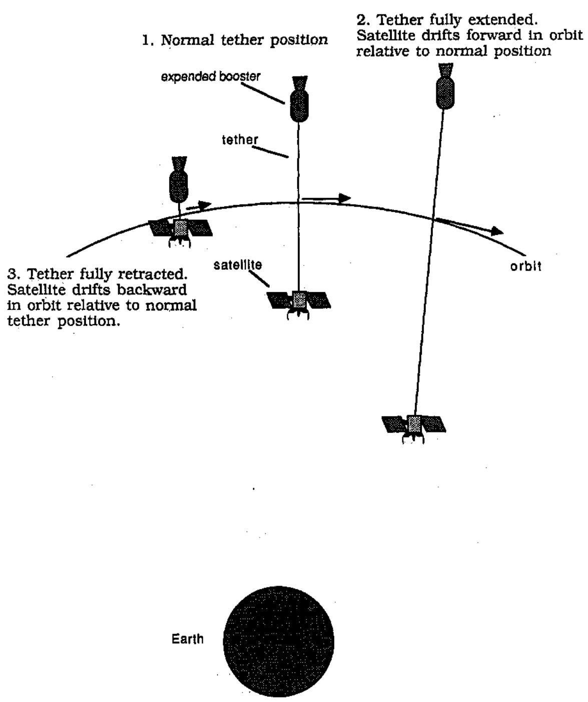

Fig. 2 Satellite relocation by extension and retraction of a tether.

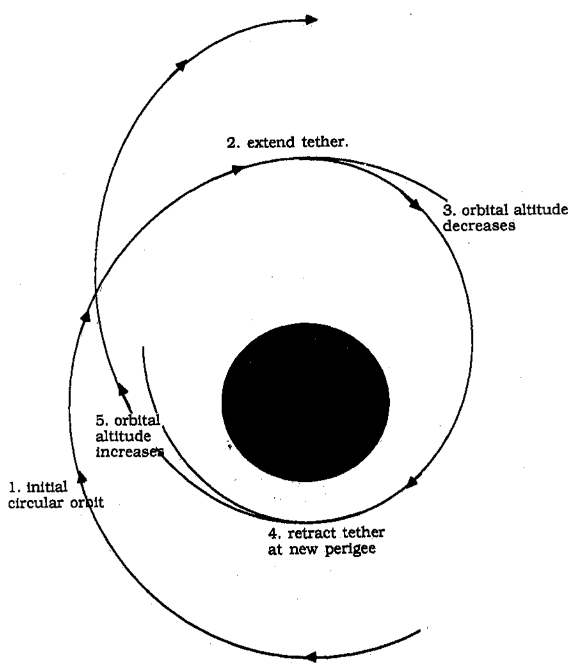

Fig. 3 Tether length variation causing eccentricity change (eccentricity greatly exaggerated).

eccentricity.[7] As shown in the schematic in Fig. 3, this occurs if the deployment rate has a Fourier component equal to the orbital period.

Energy Storage

Note that the tether need not be an otherwise inactive part of the satellite. Since energy is gained in deploying the tether out and used in pulling the tether in, it could be used for energy storage. An optimal design for a geosynchronous satellite might well use this energy storage in place of batteries to power the satellite electrical systems during the eclipse period (which lasts about 1.1 h per day for a period of several days around the equinox) when no solar energy is available to power the solar cells. The small orbital change produced by this could be easily corrected by reeling the tether in to compensate during the sunlit portion of the orbit. By integrating Eq. (1), the total energy available is

$$E = 1.5 m g L^2 r_e^2 / r_o^3 \tag{8}$$

For E, the energy in W-h, L, the half length in km, and m, the satellite mass in kg, in GEO the specific energy is 0.55 W-h/kg of satellite mass. From Eq. (2), the maximum energy that can be stored per kilogram tether mass is

$$E/m_t = 0.5\ gL_c \tag{9}$$

independent of orbital radius. For a Kevlar cable, this is about 350 W-h/kg. Even including a large safety margin, this compares favorably to the 14 W-h/kg produced by current technology Ni-H batteries.

The tether may have additional uses on the satellite as well. In addition to correction of small perturbations in the orbital period (e.g., due to lunar and solar pertubations) by adjusting the tether length, modulation of the length in phase with the orbital period could be used to adjust changes in the orbital eccentricity.[7,8] Another possible use for the tether is as an element of the initial boost of the satellite into position by the well-researched process of tether boost.[1,2] Finally, the requirement to remove the satellite from geosynchronous orbit at the end of life can be accomplished by cutting the tether at full extension.

Conclusions

A new method has been described for a satellite in orbit to be repositioned in orbital longitude by use of a tether connecting the active satellite with an inert mass, such as the expended booster. In addition to allowing repositioning, the system also allows correction of small changes in orbital period and eccentricity and use of the tether as an energy storage source.

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Attitude and Spin Rate Control of a Spinning Satellite Using Geomagnetic Field

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Introduction

THE use of magnetic control torques resulting from the interaction between the geomagnetic field and a coil magnetic moment is one of the earliest satellite attitude control methods.[4-6] This Note studies both the spin axis reorientation as well as the spin rate control of the Brazilian Data Collecting Satellite (BDCS). The solution of the spin axis reorientation problem is based on the work of Shigehara.[3] The spin rate control is performed by the appropriate switching of a plane magnetic coil.

Dynamic Modeling

The differential equations for the attitude motion of spin-stabilized satellites were developed using the spin axis spherical coordinates α (right ascension) and δ (declination) and spin rate ω. From Ref. 1, we have

$$I_z\omega \cos\delta\dot{\alpha}\boldsymbol{I} + I_z\omega\dot{\delta}\boldsymbol{J} + I_z\dot{\omega}\boldsymbol{K} = \boldsymbol{T}_1 \tag{1}$$

where I_z is the moment of inertia about the z axis (spin axis); $\boldsymbol{I}$, $\boldsymbol{J}$, and $\boldsymbol{K}$ are the unit vectors in the inertial coordinate system, and $\boldsymbol{T}_1$ is the resulting torque acting on the satellite. It is given by $\boldsymbol{T}_1 = \boldsymbol{T}_a + \boldsymbol{T}_p + \boldsymbol{T}_e$ where $\boldsymbol{T}_a$ and $\boldsymbol{T}_p$ are the torques representing the interaction between the geomagnetic field and magnetic moments along and orthogonal to the spin axis, respectively; $\boldsymbol{T}_e$ is the torque arising from the eddy currents[1]:

$$\boldsymbol{T}_a = -(m_i + m_r)(B_y\boldsymbol{i} - B_x\boldsymbol{j}) \tag{2a}$$

$$\boldsymbol{T}_p = v\,[m_p \sin\psi\, B_z\boldsymbol{i} - m_p \cos\psi\, \boldsymbol{j} + (m_p \cos\psi\, B_y - m_p \sin\psi\, B_x)\,\boldsymbol{k}] \tag{2b}$$

$$\boldsymbol{T}_e = p\,\omega\,[B_xB_z\boldsymbol{i} - B_yB_z\boldsymbol{j} + (B_x^2 + B_y^2)\,\boldsymbol{k}] \tag{2c}$$

In the foregoing expressions, m_r is the residual magnetic moment and m_i and m_p are the magnetic moments generated by the axis and plane coil, respectively; B_x, B_y, and B_z are the components of the geomagnetic field[2] in the satellite coordinate system $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$, $\boldsymbol{k}$ being aligned with the spin axis; $v \in (-1; 0; +1)$ is the polarity of the plane coil; ψ is the phase angle of the plane coil, and p is a constant depending on the satellite geometry and material conductivity.[1]

Control Law

Spin Axis Attitude Control

The spin axis can be steered between two given attitudes by conveniently switching the axis magnetic coil. The control

Received February 28, 1989; revision received May 30, 1989; accepted for publication June 16, 1989.

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